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Using the Intermeans Parameter to Model the Textile Demand : Statistical Analysis & Simulations

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the newsboy paradigm

introduction

- (Scarf's notations) y : order quantity, $\Phi(\xi)$: demand cdf, c : unit cost, r : unit selling price
- unsold units are discarded
- the satisfied demand is : $\xi^+ = \min(y, \xi)$
- naive solution : $\widehat{G} \doteq G(\mu, \mu)$... but actual gain (ex post) :

$$G(y, \xi) = r\,\xi^+ - c\,y$$

cost of a given choice

- define $\theta_y \doteq \int_y^\infty \mathrm{d}\Phi\,(\xi)$ together with:

$$\xi_y^a \doteq \mathrm{E}\,(\xi \mid y < \xi) \quad ; \quad \xi_y^b \doteq \mathrm{E}\,(\xi \mid \xi < y)$$

- define $G\,(y,\,\Phi) \doteq \mathrm{E}\,(G\,(y,\,\xi))\,\xi$, obtain:

$$\widehat{G} - G\,(y,\,\Phi) =$$

$$\theta_y\,\left(\xi_y^a - y\right)\,(r - c) + (1 - \theta_y)\,\left(y - \xi_y^b\right)\,c$$

- and conclude:

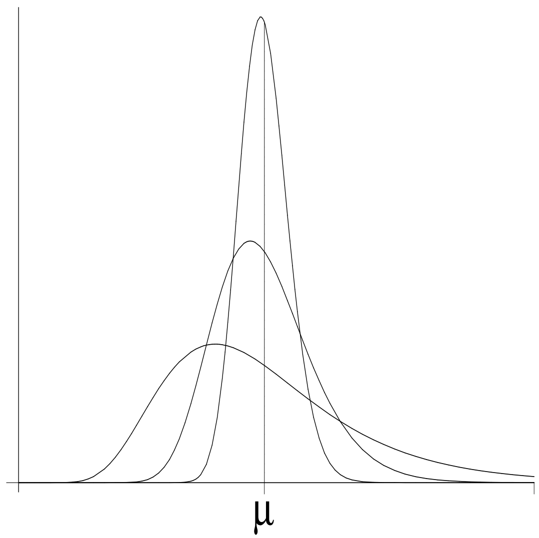
$$\forall\Phi\,\forall y\ :\ G\,(y,\,\Phi) \leq \widehat{G}$$

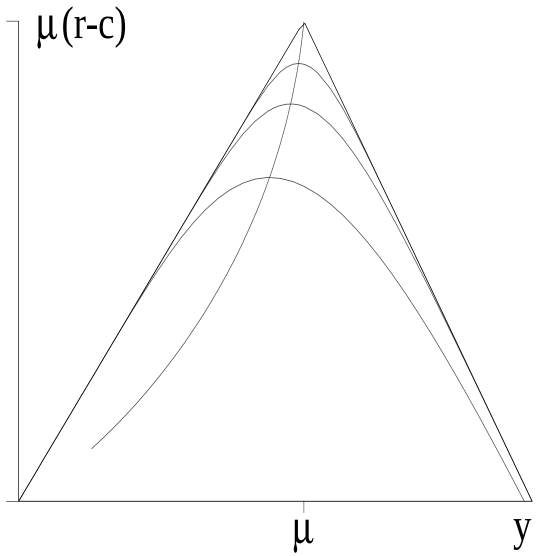
cost of uncertainties

- knowing Φ, we have $y^* = \arg\max_y G(y, \Phi)$
- exact analytical solution : $\theta_{y^*} = \theta_* = 1 - \Phi(y^*) = c/r$
- the corresponding (least) miss to gain can be rewritten as:

$$\widehat{G} - G(y^*, \Phi) = \theta_* (1 - \theta_*) \left(\xi_*^a - \xi_*^b\right) r$$

example : lognormal Φ, given μ





- positive values, but assume multiplicative independence
- curve $\sigma \mapsto (y^*, G^*)$, assuming $c/r < 1/2$

robust solutions

basic questions

- does Φ model a lack of knowledge due e.g. to their cost or model the intrinsic wild behavior of the markets ?
- is Φ guessed from many parallel independent worlds or induced from historical data (questionable ergodicity) ?
- can ξ be ever measured, even afterwards, when the demand overflows the inventory ?
- aren't we introducing abusive "side assumptions", from computing easiness rather from an actual knowledge ?

Scarf's theorem (1)

- robust solution against a family $\mathcal{F}$ of distributions:

$$G^* \doteq G\left(y^*, \mathcal{F}\right) \doteq \max_{y} \min_{\Phi \in \mathcal{F}} G\left(y, \Phi\right)$$

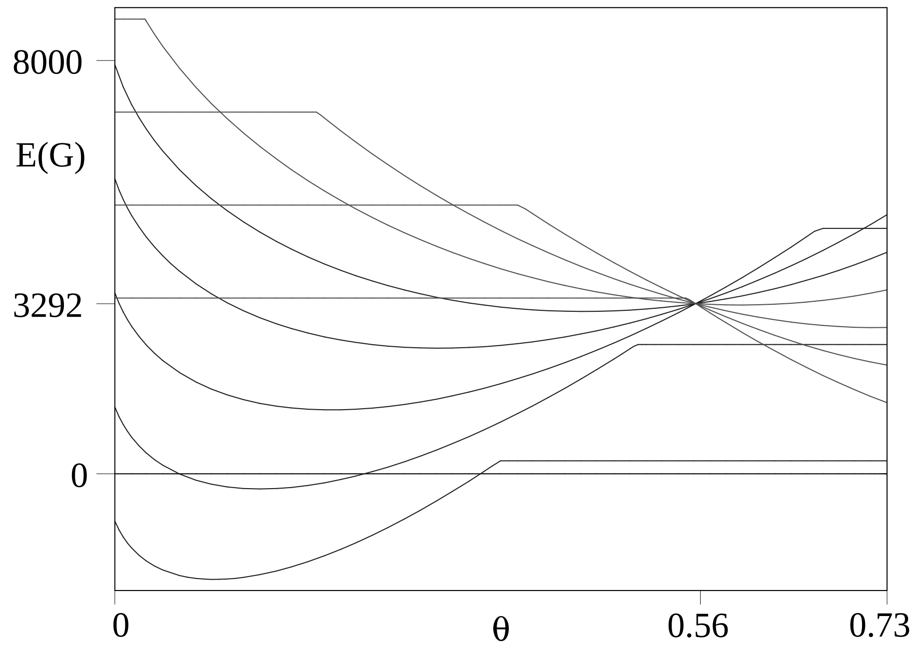
- when playing against $\mathcal{F}\left(\mu, \sigma\right)$, the worst case is a Dirac two points distribution (i.e. ξ is either ξ^a or ξ^b) and:

$$\begin{cases} y^* & = & \mu + \sigma\left(r/2 - c\right)/\sqrt{c\left(r-c\right)} \\ \\ G^* & = & \mu\left(r-c\right) - \sigma\sqrt{c\left(r-c\right)} \end{cases}$$

Scarf's theorem (2)

- graphical proof : playing against $\mathcal{F}(\mu, \sigma, Dirac)$ is:

  - chose y, i.e. a curve and wait for answer $G(y, \xi)$
  - $\mathrm{E}(G)$ depends on θ
  - all curves are concurrent at $\theta = c/r$ (here 0.56)



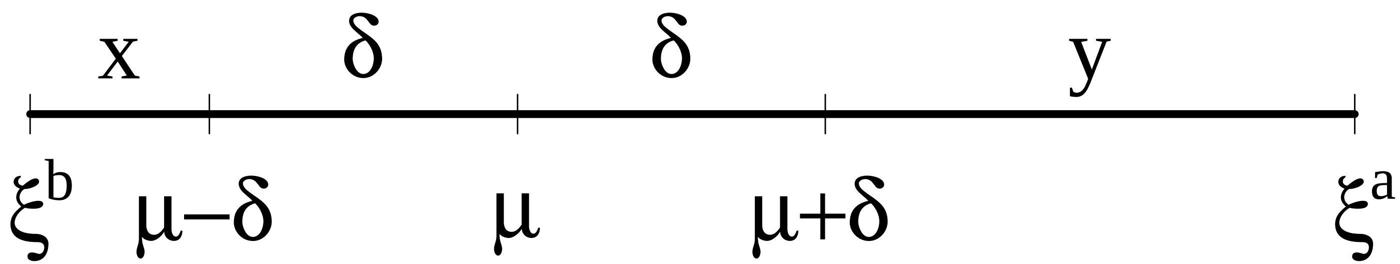
- when $(c/r)^{-1} < 1 + \sigma^2/\mu^2$ (here 0.73), better do nothing

another measure of the dispersion

- when $y = \mu$, $\widehat{G} - G\left(\mu,\, \Phi\right) = \theta_\mu \left(1 - \theta_\mu\right) \left(\xi^a_\mu - \xi^b_\mu\right) \times r$
- this kind of factorization applies only to y^* and μ
- thus $\widehat{G} - G^* \leq \delta\, r$, independent of c/r, where

$$\delta \doteq \theta_\mu \left(1 - \theta_\mu\right) \left(\xi^a_\mu - \xi^b_\mu\right)$$

- properties : $\xi^b < \mu - \delta < \mu + \delta < \xi^a$ and $x\, y = \delta^2$



information value of assumptions

numerical example

- consider a set of historical data were $y = 999$ each $n = 16$ time has resulted in the following sales:

$$743, 999, 999, 999, 851, 939, 601, 483$$
$$999, 655, 999, 856, 821, 810, 999, 999$$

- obtain the following estimators:

$$\text{est}\left(\theta_{999}\right) = 0.44 \quad ; \quad \text{est}\left(\xi_{999}^{b}\right) = 751$$

add some hints for the $y < \xi$ case

- …endure extra costs to obtain extra knowledge…
- guess the non observed demand:

$$., 1278, 1418, 1461, ., ., ., .$$
$$1176, ., 1016, ., ., ., 1194, 1028$$

- and obtain the following estimators:

$$\text{est}\left(\xi^b\right) = 751\ ;\ \text{est}\,(\mu) = 958\ ;\ \text{est}\,(\xi^a) = 1224$$
$$\text{est}\,(\sigma) = 278\ ;\ \text{est}\,(\delta) = 116$$

best decision for a given knowledge

known Φ or $\mathcal{F}$	$\frac{c}{r} = .3$ y^*	$\frac{c}{r} = .3$ G	$\frac{c}{r} = .7$ y^*	$\frac{c}{r} = .7$ G
$\Phi\,(\mu,\ \sigma,\ normal)$	1108	5710	807	1878
$\Phi\,(\mu,\ \sigma,\ lognormal)$	1117	5916	821	2069
$\mathcal{F}\,(\mu,\ \sigma,\ triangular)$	1103	5607	813	1774
$\mathcal{F}\,(\mu,\ \sigma)$ (Scarf's rule)	1083	5393	833	1560
$\mathcal{F}\,(\mu,\ \delta,\ Dirac)$	958	5541	958	1709
$\mathcal{F}\,(\mu,\ \delta,\ triangular)$	1159	5807	818	1807

value of shape when σ is known

actual	known	y^*	$G(y, \Phi)$	Δ
$\Phi(\mu, \sigma, normal)$	only μ, σ	1083	5706	
$\Phi(\mu, \sigma, normal)$	Φ	1108	5710	4
$\Phi(\mu, \sigma, lognormal)$	only μ, σ	1083	5911	
$\Phi(\mu, \sigma, lognormal)$	Φ	1117	5916	5

- when μ, σ are known, an additional knowledge of the shape has only a small value

other information values

value of using δ versus σ

actual $\mathcal{F}$	Newsboy's belief	y^*	worst θ	G	Δ
$\mathcal{F}(\mu, \sigma, Dirac)$	δ constant	958	$\theta = 0.5$	5273	
	σ constant	1083	$\theta = 0.3$	5393	120
$\mathcal{F}(\mu, \delta, Dirac)$	σ constant	1083	$\theta = 0$	5389	
	δ constant	958	*all*	5541	152

comparison δ versus σ

Φ	δ/σ(exact)	δ/σ(approx)	0.25 gap
uniform	$\sqrt{3}/4$	≈ 0.433	
triangular	$1/\sqrt{6} \cdots 8\sqrt{2}/27$	$.408 \cdots .419$	
normal	$1/\sqrt{2\,\pi}$	≈ 0.399	
exp	$1/e$	≈ 0.368	
Dirac	$\sqrt{\theta\,(1-\theta)}$	$0 \cdots 0.5$	$\theta = 7\%,\ 93\%$
lognormal	$\leq 1/\sqrt{2\,\pi}$	$0 . \cdots 0.399$	$\sigma/\mu \approx 2$

in all realistic situations : $0.25\,\sigma \leq \delta \leq 0.50\,\sigma$

uncertainties...

- in fact, est (μ) is roughly normal, with variance $\sigma^2/n \approx 5135$
- est $\left(\sigma^2\right)$ is roughly normal, with variance $\left(M_4 - \sigma^4\,(n-3)\,/\,(n-1)\right)/n \approx 4.289\,10^8$, where M_4 is the fourth centered moment.
- In other words, our actual knowledge is :

$$\mu \in [958 \pm 72\,k_\mu] \quad ; \quad \sigma^2 \in [82166 \pm 20710\,k_\sigma]$$

- $k \leq 2$ is an optimistic choice (at the 95% level)

value of parameters

k_μ	actual μ	guessed μ	y^*	θ^*	$G(y, \Phi)$	Δ
-2	814	958	1083	0.16	4277	
		814	939	0.30	4390	113
$+1$	1030	958	1083	0.41	5857	
		1030	1155	0.30	5895	38
$+2$	1101	958	1083	0.53	6237	
		1101	1227	0.30	6397	159

conclusion

- the best decision for the newsboy problem depends heavily on the choice of the dispersion measure
- the well known "Scarf's rule" follows when assuming an exact knowledge of μ, σ
- but another strategy follows when assuming an exact knowledge of μ, δ
- this happens while $0.3 \leq \delta/\sigma \leq 0.5$ for all relevant Φ and while estimator d doesn't behave worse than estimator S

- an exact identification of reduced parameters concerning the demand models seems to be questionable
- extraction of knowledge from history cannot be model-free (ξ^a is beyond any experience)
- a description using larger families of pdf such as $\mathcal{F}(\mu \pm \Delta\mu, \sigma \pm \Delta\sigma)$ or $\mathcal{F}(\mu \pm \Delta\mu, \delta \pm \Delta\delta)$ seems a better way for a robust description