GEMTEX

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How Robust is a Newsboy Model ?

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- probability distributions are often used to express a limited knowledge
- too often, side assumptions are introduced that are not founded on that actual knowledge, but only on computational facilities
- the robustness of the conclusions drawn must be checked !

robustness is a key concern

- Scarf's notations

  y : order quantity, $\Phi(\xi)$: demand cdf,

  c : unit cost, r : unit selling price

  satisfied demand $\xi^{+} = \min(y, \xi)$

- non sold units are discarded

  actual gain $G(y, \xi) = r\xi^{+} - cy$

the newsboy paradigm

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- naive solution $G_0 \doteq G(\mu, \mu)$
- expected gain

$$G(y, \Phi) = E(G(y, \xi)) =$$

$$-cy + r\left(\int_0^y \xi \, \mathrm{d}\Phi(\xi) + y \int_y^\infty \mathrm{d}\Phi(\xi) \right)$$

- analytical solution $\Phi(y^*) = 1 - \dfrac{c}{r}$

a well known formula

- define $\theta \doteq \int_y^\infty \mathrm{d}\Phi(\xi)$ and ξ_{left}, ξ_{right} by

$$\xi_l \doteq \frac{1}{1-\theta}\int_0^y \xi \,\mathrm{d}\Phi(\xi) \text{ and } \xi_r \doteq \frac{1}{\theta}\int_y^\infty \xi \,\mathrm{d}\Phi(\xi)$$

- and obtain

$$G_0 - G(y,\Phi) = (1-\theta)(y-\xi_l) + \theta(\xi_r - y)$$

- thus $\forall \Phi : \forall y : G(y,\Phi) \leq G_0$

the cost of uncertainties

using different models

- normal
- lognormal
- triangular
- “two Diracs (Scarf’s model)”

and the following parameters

- fixed $\mu = 1000,\ c = 12, r = 20$
- variable σ/μ (namely $0, .1, .2, .4$)

a comparative study

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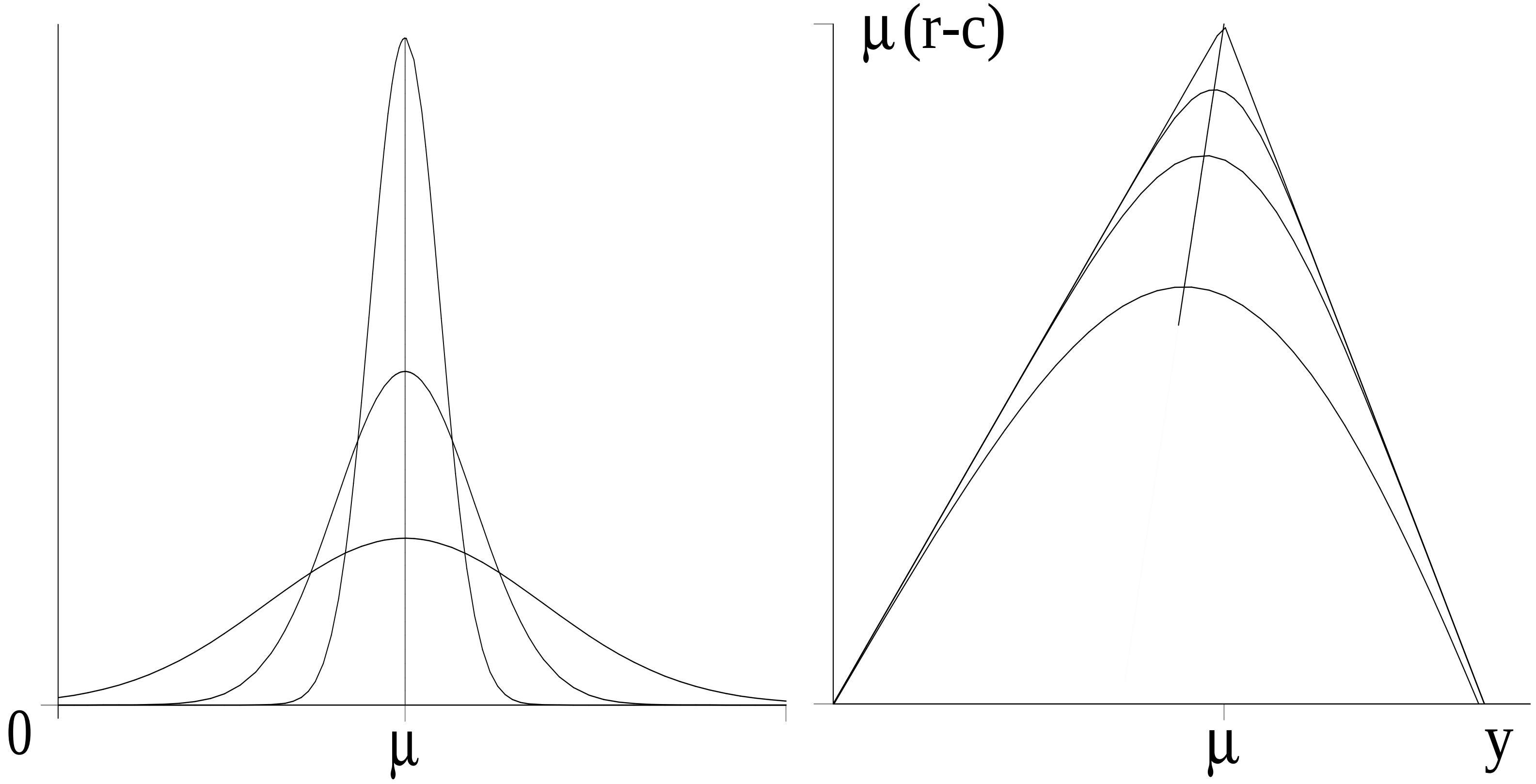

- additive independence (consumers ?)
- necessitates small values of σ/μ

normal model

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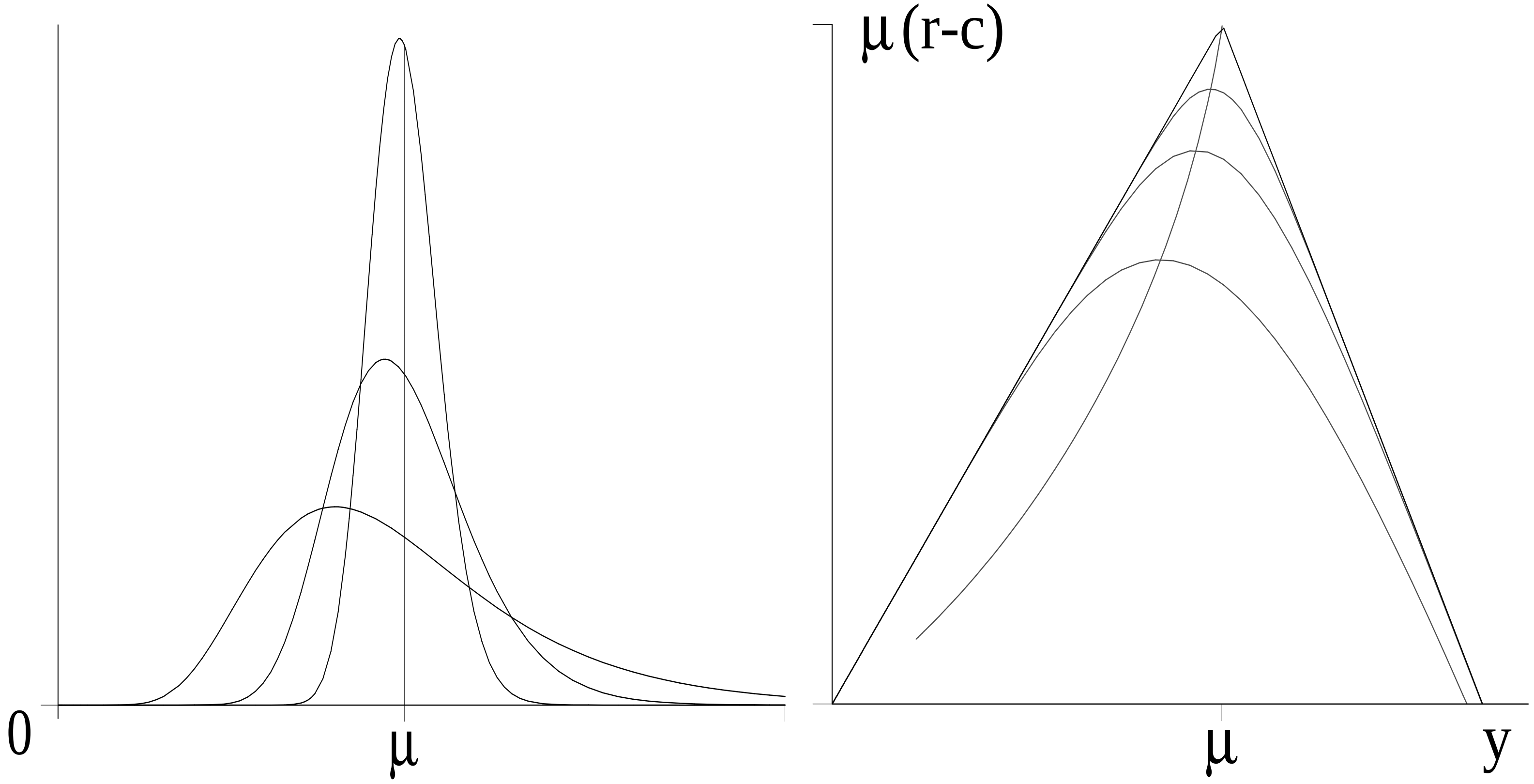

- multiplicative independence (atmospherics ?)
- special shape of the maximum locus

lognormal model

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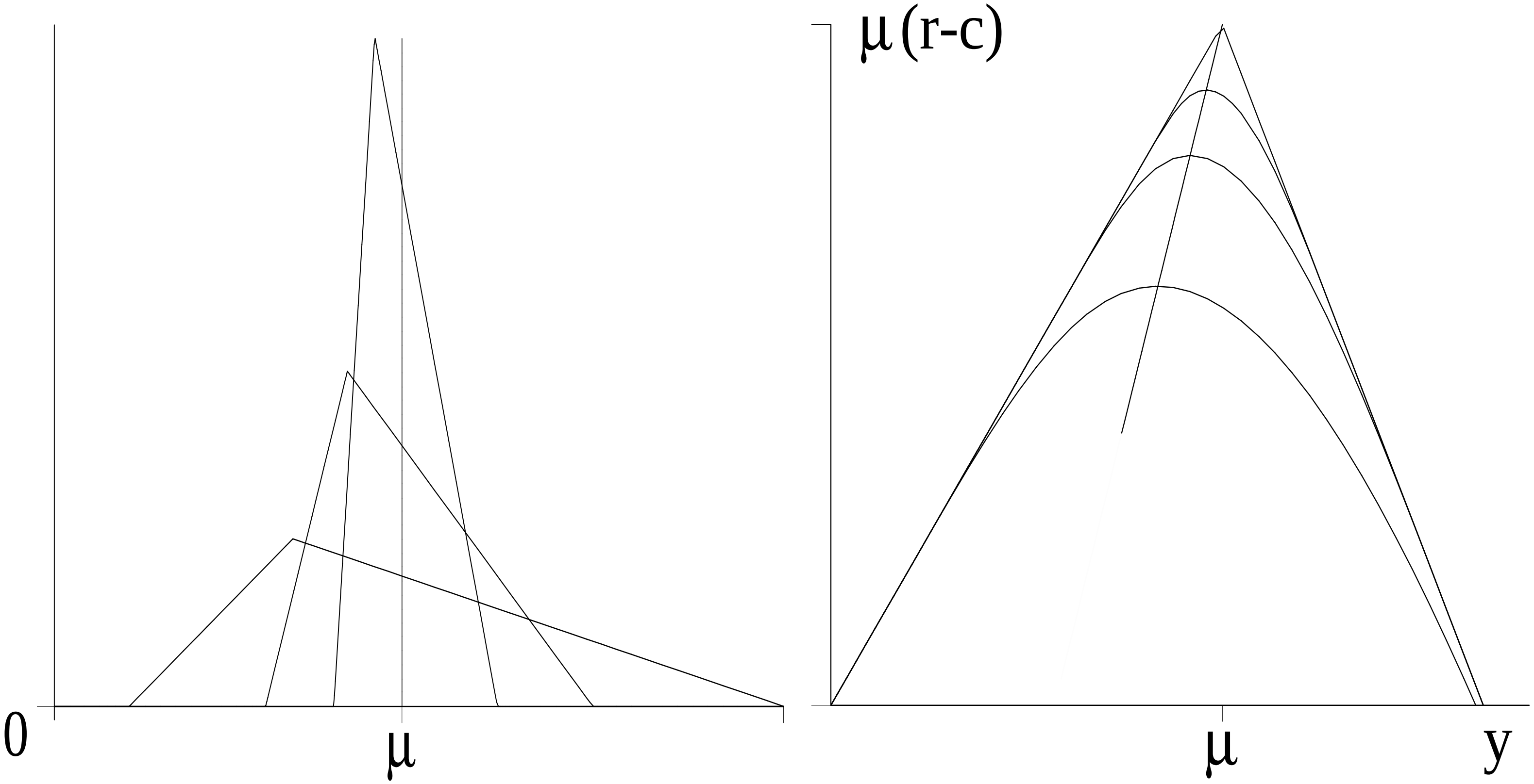

- positive values, three parameters, easy to use
- have you a knowledge against that model ?

triangular model

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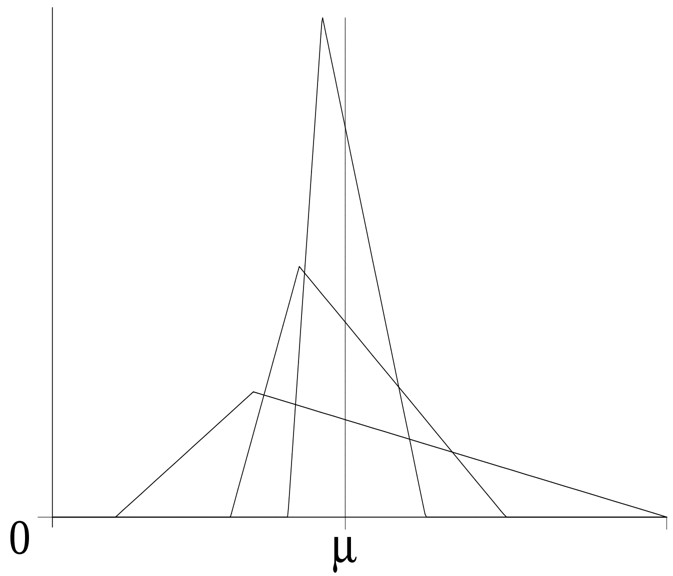

$$\Phi(\xi) = \begin{cases} \dfrac{(\xi-\alpha)^2}{(\gamma-\alpha)(\beta-\alpha)} & \xi \leq \beta \\ \dfrac{(\gamma-\xi)^2}{(\gamma-\alpha)(\gamma-\beta)} & \xi \geq \beta \end{cases}$$

$$\tau = -1 + 2(\beta-\alpha)/(\gamma-\alpha)$$

- $\mu = \frac{1}{3}(\alpha+\beta+\gamma), \quad \sigma^2 = \frac{1}{36}\left((\gamma-\beta)^2+(\beta-\alpha)^2+(\alpha-\gamma)^2\right)$

- skewness $\sigma = \frac{\sqrt{2}}{12}\Delta\sqrt{3+\tau^2}, \; M_3 = \frac{-1}{1080}\Delta^3\tau\left(9-\tau^2\right)$

several formulae

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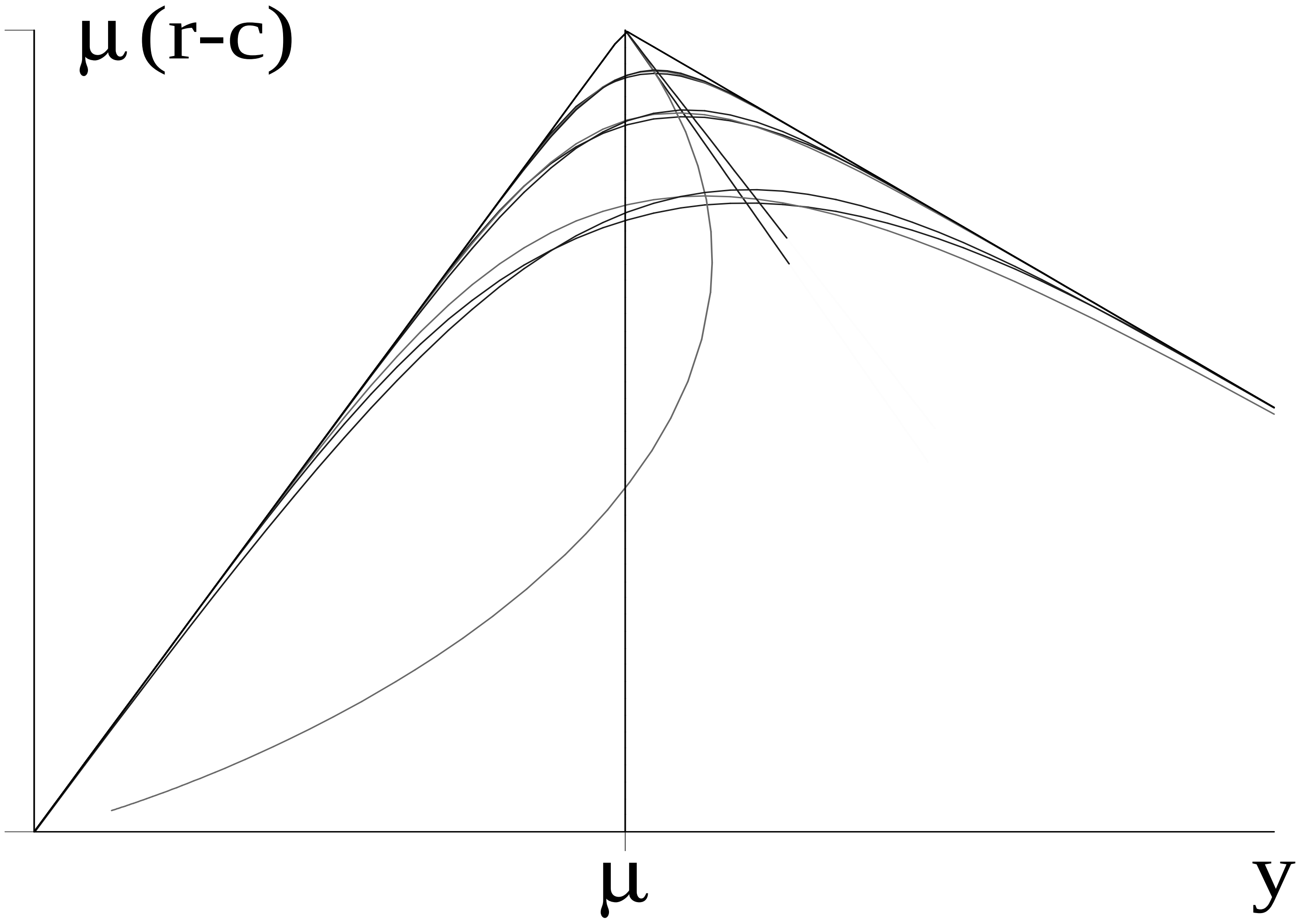

when c/r < 1/2

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- does Φ model a lack of knowledge due e.g. to their cost or model the intrinsic wild behavior of the market ?
- is Φ an average over all the many parallel independent worlds or is Φ induced from an assumed ergodic property of historical data ?
- can ξ be ever measured, even afterwards, when the demand overflows the inventory ?

discussion about hypotheses

- when $y=\mu$, $y=(1-\theta)\xi_l+\theta\xi_r$ holds

- when $y=\mu$, $G_\mu \doteq G(\mu,\Phi)$ verifies

$$G_0 - G_\mu = r\,\theta(1-\theta)(\xi_r - \xi_l)$$

- since $\theta = \Pr(\xi>\mu)$, $\xi_l = \mathrm{E}(\xi \mid \xi<\mu)$ and $\xi_r = \cdots$ the quantity

$$\delta \doteq \theta(1-\theta)(\xi_r - \xi_l)$$

is a measure of the dispersion of the demand

the naive and obstinate merchant

$$\delta \doteq \theta(1-\theta)(\xi_r - \xi_l)$$

distribution	δ/σ exact	δ/σ approx
uniform	$\sqrt{3}/4$	0.433
normal	$1/\sqrt{2\pi}$	0.399
lognormal	$< 1/\sqrt{2\pi}$	< 0.399
triangular	$1/\sqrt{6}$ .. $8\sqrt{2}/27$	0.408 .. 0.419
general		≤ 0.5 ?

the "inter-mean" interval

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- Scarf's functions

$$d\Phi(\xi) = (1-\theta)\,\mathrm{Dirac}(\xi-\alpha) + \theta\,\mathrm{Dirac}(\xi-\gamma)$$

- over all the Φ that shares the same μ, σ, the worst distribution against a given order quantity y is a Scarf's function

- thus $G_{robust} = \max_y \; \min_{\Phi|\mu,\sigma} \; G(y, \Phi)$ is

$$\begin{cases} \text{if } \dfrac{c}{r}\left(1+\dfrac{\sigma^2}{\mu^2}\right) < 1 \text{ then } & y_{robust} = \mu + \sigma \dfrac{r/2 - c}{\sqrt{c(r-c)}} \\ \text{otherwise} & y_{robust} = 0 \end{cases}$$

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recalling the Scarf's bound

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- Scarf's max-min using fixed μ, σ
- max-min using fixed μ, δ

$$\mu = 1000\,,\ \sigma = 600\,,\ \delta = 300$$

$$c/r = 5/9\,,\ c - r = 10$$

comparison

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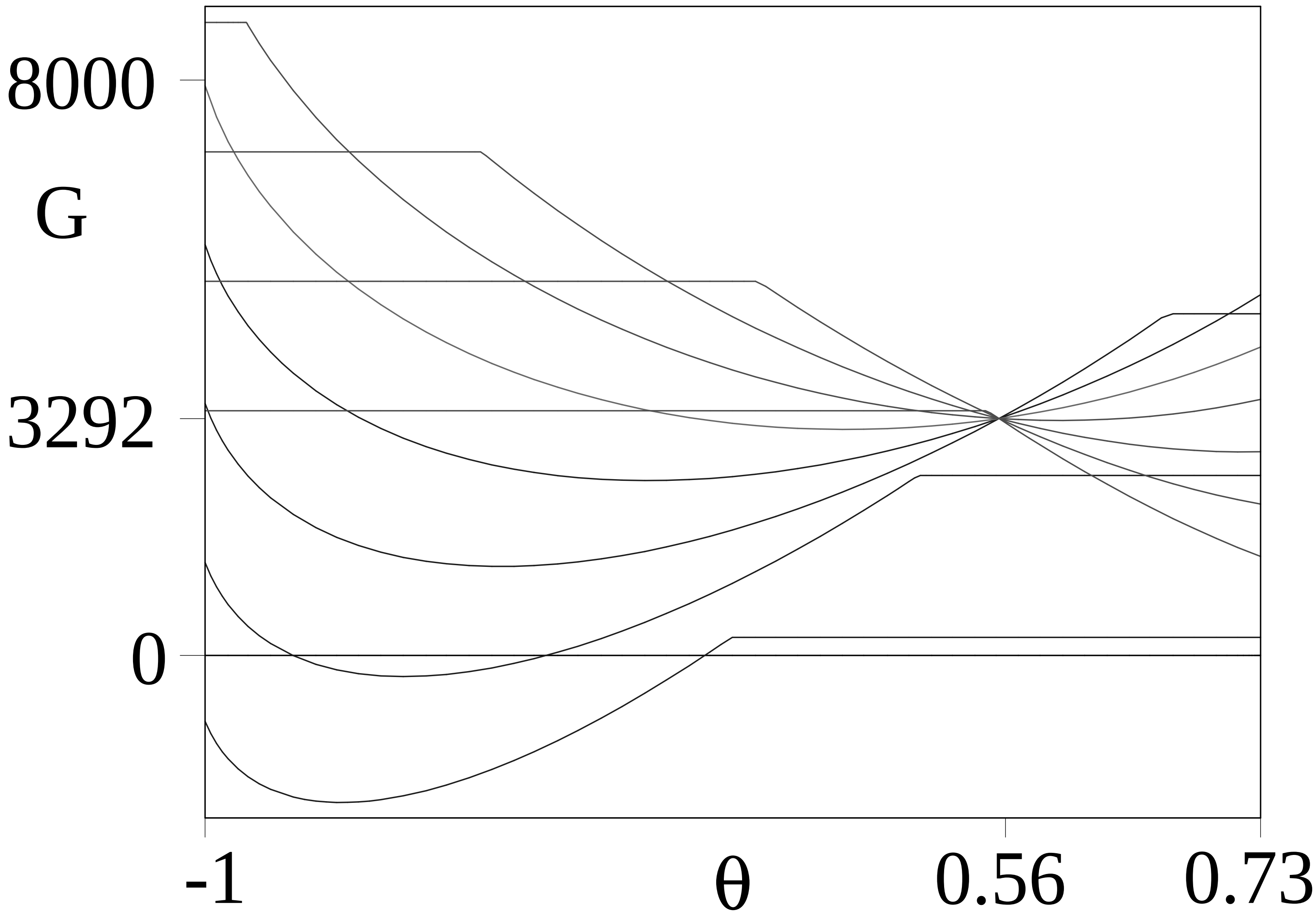

graphical proof of Scarf's theorem

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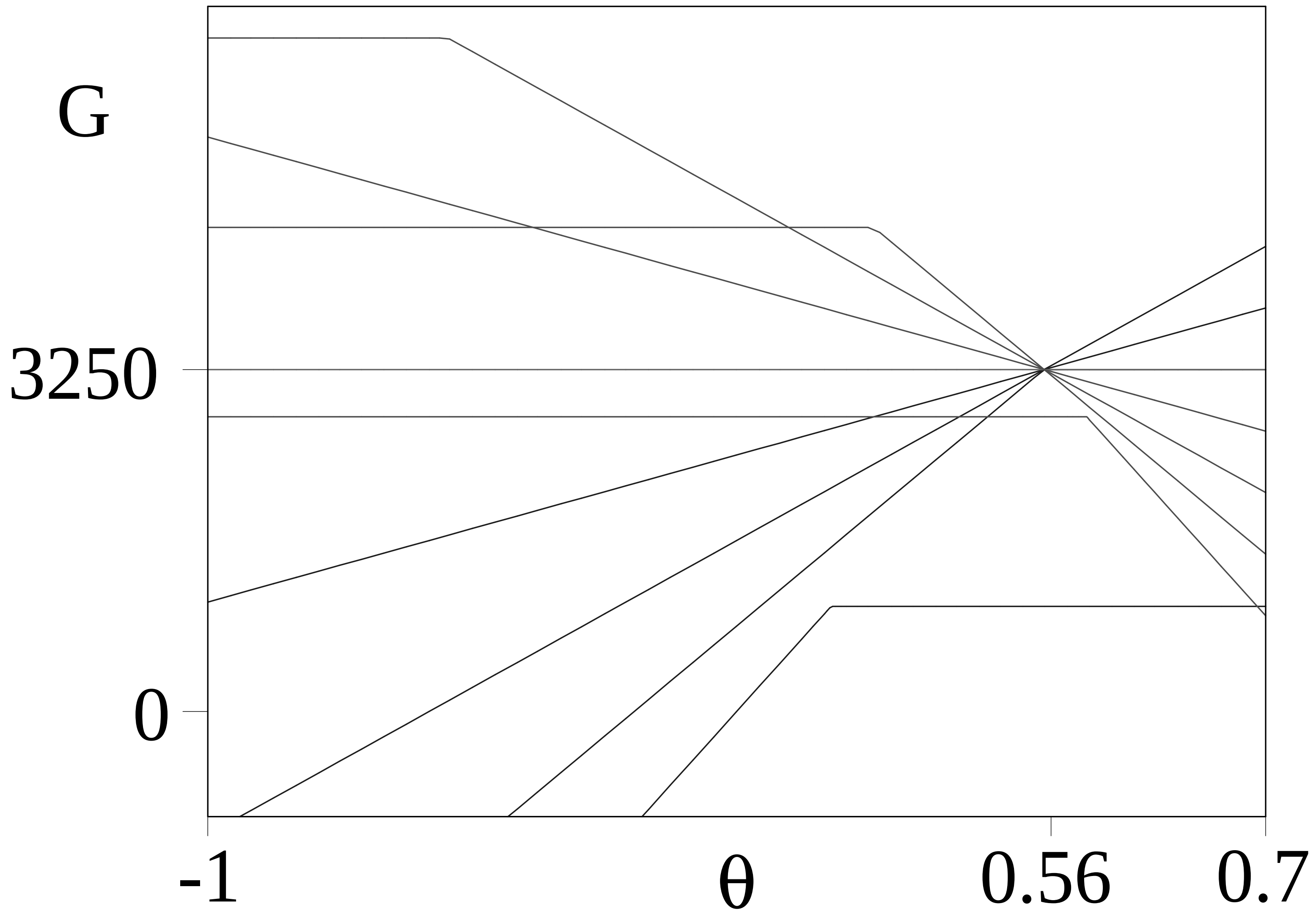

using the same method

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$$\delta \doteq \theta(1-\theta)(\xi_r - \xi_l)$$